JOURNAL OF APPLIED POLYMER SCIENCE VOL. 7, PP. 2105-2120 (1963)

Friction and Deformation of Nylon. II. Theoretical

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1. INTRODUCTION

The adhesion theory of friction interprets the friction between two bodies as the force required to shear their interface. This force is taken to be the product of the area of true contact between the bodies and its specific shear strength S_T. Usually, for simplicity, S_T is assumed to be constant but the possibility that it can be pressure-dependent has also been considered and molecular theories have been advanced which predict this, e.g., those of Deryaguin[1] and Kraghelsky.[2]

Theories of the friction of rough elastic bodies are therefore primarily methods of estimating the load dependence of the true contact area between their surfaces. The basic problem here is the representation of the roughness of the bodies in a form which makes this estimation possible and which is a plausible model of a real surface.

The difficulty in performing precise friction measurements on highly elastic materials and in measuring sufficient parameters to specify their surfaces has made the possibility of a crucial test of such theories seem remote. In fact, it has not been necessary to assume pressure dependence of S_T to interpret available data on these materials, so that this parameter of the theories has remained in reserve.

In the preceding paper the friction F and the apparent contact area A of nylon hemispheres sliding on glass were shown to be related to the load W by equations of the form $F = \alpha W^n$ and $A = \beta W^m$ where the values of n and m were 0.781 and 0.708, respectively. In this paper it is shown that the size of $n - m$ implies that a pressure dependence of S_T is necessary. In order to do this the published friction theories of rough elastic bodies are outlined to demonstrate their inability to explain the observed value of $n - m$, assuming a constant value of S_T. This limitation is shown not to be removed by the assumption that under the pressures existing within the apparent contact area some asperities are completely flattened, without the simultaneous assumption of a pressure-dependent value of S_T. This last assumption is therefore considered to be essential.

A theoretical treatment of the effect of complete flattening of some of the asperities within the contact leads to the prediction of a change of slope of log F plotted against log W at a critical load, which may be estimated from

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practicable roughness measurements. Comparison with experiment is satisfactory.

2. THEORIES OF THE FRICTION OF ROUGH ELASTIC BODIES

Published treatments, e.g., those of Lodge and Howell,[3] Rubenstein,[4] and Archard,[5,6] have assumed that the roughness is expressible by asperities in the form of hemispheres or hemispherical caps whose lateral dimensions are small compared with those of the apparent contact area, and they have calculated the true contact area from the elastic deformation of the numerous asperities within that area. All treatments have assumed that the same deformation law is obeyed by all the asperities within the apparent contact area at all loads. The choice of hemispherical shape for the asperities is justified by the geometrical theorem that over sufficiently small regions any convex surface can be described by two principal curvatures and the fact that, at least as far as the load dependence of contact area is concerned, no loss of generality is involved in assuming the curvatures to be equal. On physical grounds one might expect surface forces in polymers prepared from solutions or melts or by polishing to cause any asperities to have a rounded form.

For simplicity we will only consider the contact between one surface covered with asperities and one rigid, smooth, plane surface. This resembles the conditions in the experiments on the friction of nylon specimens on a polished glass plate.

The theoretical treatments differ in the assumed arrangement of the asperities. Lodge and Howell, and Rubenstein, assume the asperities to be of constant height so that their tips lie in a surface similar to that of the substrate; e.g., on a plane substrate the tips of all asperities are coplanar. Archard, on the other hand, assumes the asperities to be of various heights so that the number of asperities intersected by a surface of uniform separation from the substrate will increase as this separation decreases.

The methods of calculating the true contact area also differ. Lodge and Howell assume the load to be shared among the asperities within the apparent contact area so that the load on each asperity is proportional to the area of its base multiplied by the pressure that would be observed at its corresponding position if the surfaces were both smooth. In effect, the load on an asperity is determined by the deformation of the substrate. The total true contact area obtained by summing the contact areas of the individual asperities under these loads is then related to the total load.

Archard assumes that the substrate on which the asperities are based is rigid so that when the contacting bodies approach by a distance x, from being just touching, the tips of all asperities which were less than x below the highest asperity will be partially flattened. It is thence possible to calculate, from an assumed height distribution of asperities, the load and contact area of the individual asperities as functions of x. The dependence of the total true contact area on the total load is obtained by summing these individual loads and contact area and then eliminating x.

For neither Lodge and Howell's nor Archard's model does the curvature of the asperities affect the load dependence of true contact area, provided that there is no correlation of asperity curvature with position on the surface or with asperity height. Both models predict the true contact area to depend on a power of the load.

General expressions for the index n of the relation $F = \alpha W^n$ given by the two theories are given in Table I for hemispherical bodies against flat surfaces. The effect of a possible dependence of the shear strength of an element of the true contact area on the normal pressure on that element is included. It has been assumed that the asperities deform with load according to the relation $A' = \beta' W^{m'}$ where m' could differ from the value m for a specimen. The expression based on Archard's theory was calculated by the writer and is in close agreement with the particular values given by Archard.

TABLE I
Theoretical Values of the Friction Index n[a]

	n (Lodge and Howell)	n (Archard)
Rough hemisphere on plane	$m + (1 - m)[m' + h(1 - m')]$	$\left(\dfrac{g + h(1/m - 1) + 3}{g + 1/m + 2}\right)$
Smooth hemisphere on plane	$m + h(1 - m)$	—

[a] Here m and m' are the indices of the area/load relations $A = \beta W^m$ and $A' = \beta W^{m'}$ which refer, respectively, to the specimen and asperity deformations, h is the index of the specific friction/pressure relation $S_T = S_0 p^h$, g is the index of the asperity frequency/height relation $N = N_0 x^g$.

From Table I we see that, since m is less than unity and the lowest physically plausible values of h and g are zero, the lowest values of n will be obtained when h and g are both zero. Table II gives these values of n when m and m' are equal and take the values $^2/_3$ (the value for a Hookean body) and 0.708 (the value obtained for the nylon specimens).

TABLE II
Minimum Theoretical Values of the Friction Index n

	m and m'	n (Lodge and Howell)	n (Archard)
Rough hemisphere	$^2/_3$	0.89	0.86
on plane	0.708	0.91	0.88

We see that, for a hemispherical specimen, both theories give very similar values of n, showing the insensitivity of n to the detailed assumption. The assumptions do, however, determine which of the theories is appropriate in given experimental conditions. Archard's assumption of a rigid substrate requires that the approach of a specimen to the plane on which it rubs is less than the height of the highest asperity, but Lodge and Howell's neglect of asperity height variations requires that the approach is large compared with this variation. Now, in the measurements on nylon specimens the approach ranged from 0.07 μ to 13 μ and the height variations of the surfaces were no greater than $\lambda/20$ or 0.027 μ, as was shown by the interference fringes. Lodge and Howell's theory is therefore the more appropriate for these specimens and loads.

The average experimental value of n for the nylon specimens, 0.781 lies between the lowest theoretical value for a rough hemisphere, ca. 0.90, and the theoretical value for a smooth hemisphere which equals m, 0.708 if h is zero. The obvious interpretation of this is that the surfaces of the specimens were effectively smooth, under the experimental conditions used, but that h has the value required to make n equal to 0.781. Before this is accepted we must exclude the alternative possibility that the other assumptions of Lodge and Howell's treatment can be altered sufficiently to account for the observed values of n.

2.1. Modifications of the Lodge and Howell Theory to Explain the Experimental Value of n

The assumption which has the most significant effect on the theoretical values of n is the assumed dependence of the contact area of an asperity on the load it bears. This may be more readily seen if we introduce the parameters $\bar{S}$, the average tangential traction on the apparent contact area, and $\bar{P}$, the average pressure on this area. Assuming that $F = \alpha W^n$ and $A = \beta W^m$ for the macroscopic properties of a specimen, we obtain for the relation between $\bar{S}$ and $\bar{P}$:

$$\bar{S} = \alpha\beta^{-[(1-m)/(1-n)]}\bar{P}^{(n-m)/(1-m)} \quad (1)$$

where $\bar{S} = F/A$ and $\bar{P} = W/A$.

Now, from the value for n according to Lodge and Howell's theory (see Table I), the index of $\bar{P}$, $(n - m)/(1 - m)$, equals $m' + h(1 - m')$. So, if h is zero, then $\bar{S} \propto \bar{P}^{m'}$ where m' is the deformation index of an asperity. Experimentally we find $n = 0.781$ and $m = 0.707_5$ whence $\bar{S} \propto \bar{P}^{0.25}$, so that if S_T is independent of pressure, i.e., if $h = 0$, we require $m' = 0.25$. This is much less than the deformation index observed for the macroscopic specimens and then the value $^2/_3$ predicted for hemispherical Hookean asperities. The possibility that this is a consequence of a different asperity shape or of the large deformations experienced by the asperities within the contact area is considered below.

2.11. Large Deformations

The deformation of an elastic hemisphere pressed against a rigid plane is adequately described by the ratio of a, the contact radius, to R, the radius of curvature of the hemisphere. If the asperities within the contact area are assumed to be close-packed hemispheres and to have the elastic properties of the bulk material, it can be shown that the ratio a/R for them is

$\leqslant 0.2$ under the loads used in the present experiments. Demonstrations that the deformation index m remains unchanged when a/R increases to values greater than 0.2 are given by several model experiments. Schallamach[7] (using rubber spheres) found that m was still $^2/_3$ although a/R increased to 0.8, and Pascoe and Tabor[8] (indenting plane nylon surfaces with steel balls) found $m = 0.74$ for a/R of 0.1–1.0. In the preceding paper it was found (for nylon hemispheres) that $m = 0.71$ for a/R of 0.006–0.1.

If the asperities were more widely spaced the number of asperities supporting the load would be fewer, and slightly higher values of a/R would be produced. However, this would involve a reduction in the ratio of true to apparent contact area, which is already improbably small. The observed tangential traction on the apparent contact area which reached 2 kg. mm.^{2} implies, even with close packing and $a/R = 0.2$, a value of 22 kg./mm.^{2} for S_T which is greatly in excess of the published estimates of the bulk shear strength of nylon: e.g., 1.5 kg./mm.^{2} of Pascoe and Tabor[8] and 6 kg./mm.^{2} of Shooter and Tabor.[9] This indicates a further criterion that asperity models must satisfy.

2.12. Asperity Shape

If the profile of a rotationally symmetric asperity is $y \propto |x^k|$, then, as k takes increasing positive integral values, we obtain asperities which are conical, paraboloidal (approximating near the origin to the surface of a sphere), and so on.

Experiments by the author on rubber cones pressed against a glass surface showed that for $k = 1$, m also is 1. It has been shown theoretically by Love[10] and experimentally by Sabey[11] that m is 1 when a rigid cone indents a plane surface of an elastic body. It follows from this result that an elastic body whose asperities are conical will obey Amontons' laws when sliding on a smooth rigid plane.

For a paraboloidal (or hemispherical) Hookean asperity we have, of course, $m = {}^2/_3$ as was shown theoretically by Hertz.[12] Experimental confirmation has been provided by, for example, Schallamach who used rubber balls.

A theoretical solution valid for any even integral value of k has been given by Steuermann.[13] He showed that m is then $2/(k + 1)$ so that the asperity profile would have to be $y \propto x^8$ if m, the asperity's deformation index, were to be near 0.25. If the $n - m$ difference were to be explained in terms of asperities of this profile, their deformation index of 0.25 would have to persist up to the highest pressures which occurred at the contacts, i.e., $\sim$6.5 kg./mm.^{2}, and this would require that the ratio of height to diameter of the asperities be $> {}^1/_{35}$ (as can be shown from Steuermann[13]).

If the asperity heights are no greater than 0.027 μ, as the interference fringes suggest, then the asperity diameters must be $<1\ \mu$. There does not appear to be any method by which the shape of asperities of such small dimensions could be measured, so that to postulate their presence on the specimens, although it would provide a formal interpretation of the $n - m$ difference, offers at present no prospect of experimental test and should only be done as a last resort.

We conclude that the experimental value of $n - m$ cannot be interpreted by the Lodge and Howell theory (except with the unverifiable assumption just discussed) if S_T is independent of pressure and the asperities obey the same deformation law at all pressures.

3. THEORY OF TRANSITION FROM ROUGH TO SMOOTH SURFACE

Because the experimental value of n lies between those obtained theoretically (when S_T is independent of pressure) for rough and smooth surfaces, it would seem reasonable to expect such a value if some of the asperities within the apparent contact area were completely flattened and the others were not. Analysis shows, however, not an intermediate value of n, but (as we will show first for a special model and then more generally) the surprising consequence that the dependence of friction on load up to a critical load is the one for a rough specimen and at greater loads is the one for a smooth specimen.

An advantage of this model is that it can lead to a greater ratio of true to apparent contact area and hence could require a smaller value of S_T than a model in which asperities were not completely flattened, thus meeting the objection raised in §2.11. However, we shall see that it does not enable the experimental value of n to be interpreted without the assumption of a pressure dependence of S_T.

For concreteness we will discuss first the contact between a rigid plane and a hemispherical Hookean specimen which is covered with identical asperities in the form of spherical caps arranged in a square lattice of side equal to the asperity diameter (see Fig. 1).

It can be readily shown using (under extreme conditions) Hertz's equation for contact radius [eq. (1) of the preceding paper] that the pressure p_0, averaged over the square of the lattice, at which one of these asperities will be completely flattened (i.e., at which $a_A = C$) is given by

$$p_0 = (C/3R_A)[E/(1 - \sigma^2)] \tag{2}$$

Geometrically, C/R_A, which is the ratio of base radius to radius of curvature of an asperity, equals sin θ, where θ is the maximum inclination of the surface of an asperity to the envelope of the specimen's surface. The ratio C/R_A was used as a roughness parameter in Rubenstein's theory[4] where under small deformations it affected the coefficient α of the friction equation $F = \alpha W^n$. Eq. (2) shows the importance of this ratio in determining the pressure at which a surface, whose roughness is specified in this way, becomes effectively smooth.

If the load W on the specimen is continuously increased from zero, a load W_0 will be reached at which the pressure at the center of the apparent contact area (which is the greatest pressure) will reach the value p_0 required to flatten an asperity. For loads greater than W_0 the pressure at the center will exceed p_0 and there will be a region of complete true contact bounded by the circle at which the pressure equals p_0. This circle will grow with further increase of load.

Now, it is shown in Appendix I that the area of the annulus between this circle and the boundary of the apparent contact area is independent of the value of the load, provided, of course, that the load exceeds W_0. This implies that the increase of apparent contact area with load beyond W_0 is entirely accounted for by the increase of true contact area in the central zone. Furthermore, it is also shown in Appendix I that within this annulus the apparent contact area subjected to pressures in the range p to

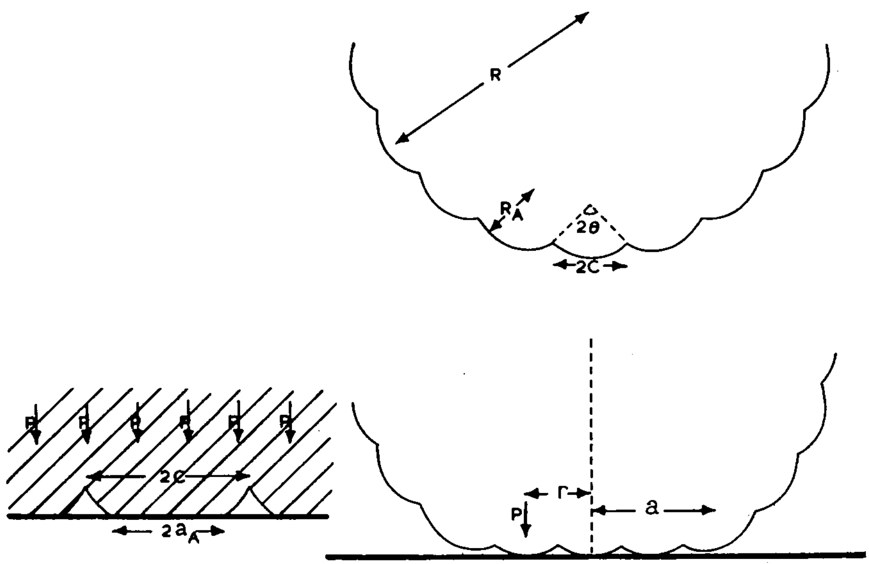

Fig. 1. Model of surface roughness.

$p + \delta p$ (where p is any value between zero and p_0) is also independent of the load, so that the contribution to the total friction by this annulus is independent of the load. The increase of friction observed as the load increases from W_0 must, therefore, be entirely accounted for by the friction of the central zone of true contact.

In the load range below W_0, i.e, when no asperities are completely flattened, the dependence of friction on load will be that predicted for rough surfaces by the theory of Lodge and Howell[3] (see S2), which is that F is proportional to $W^{8/9}$ when the shear strength of the true contact area is assumed constant. At loads greater than W_0, in view of the conclusion of the preceding paragraph, we may express the friction as follows:

$$F = F_0 + S_T(A - A_0) \qquad (3)$$

where F and A, F_0 and A_0, are the friction and apparent contact areas at loads W (greater than W_0) and W_0, respectively, and S_T is the specific shear strength of the true contact area.

Alternatively, expressing the areas in terms of the corresponding loads from the equation $A = \beta W^{2/3}$, we have:

$$F = F_0 + S_T\beta(W^{2/3} - W_0^{2/3}) \tag{4}$$

On writing this equation in the form

$$F + \Delta = S_T\beta W^{2/3} \tag{5}$$

where

$$\Delta = S_T A_0 - F_0 \tag{6}$$

we see that when F is much greater than Δ, F approaches equality with $S_T\beta W^{2/3}$. Now, it follows from eq. (6) that Δ is the shear strength of an area equal to the difference between the apparent and the true contact areas produced by the load W_0. For the special model considered here this difference can be shown to be $(1 - 3\pi/16)A_0$, i.e., 0.41 A_0, from which we derive $\Delta = 0.7F_0$. Consequently, if F exceeds 10Δ (i.e., $7F_0$), which occurs when W exceeds $8.5W_0$, deviations of F from $S_T\beta W^{2/3}$ are less than 10%.

The transition from a rough surface to a smooth surface with increasing load does not, therefore, for this model lead to a value of n intermediate between those for rough and smooth surfaces, but to a change of n from the former to the latter over a restricted load range commencing at the critical load at which asperities at the center of the contact are completely flattened.

This result is not peculiar to the assumed model, since its essential requirements are the existence of a critical pressure at which the asperities will flatten completely and the constancy of the annulus of apparent contact area subject to pressures less than this. The first requirement would be satisfied even if there were asperities of different shapes and sizes on the specimen surface, provided that elements of the surface which are small compared with the apparent contact area contain similar assortments of asperities. The second requirement is shown in Appendix I to be true for a Hookean material and also for a material which deforms according to the equation $A = \beta W^m$ when $m \neq {}^2/_3$.

We conclude that a relationship $F = \alpha W^n$ with a value of n between those predicted for rough and smooth surfaces, and which extends over a large load range, cannot be accounted for if we retain the assumption that the specific shear strength of the true contact area is independent of pressure.

3.1. Pressure-Dependent Shear Strength

The consequences of the assumption of a dependence of specific friction S_T of an element of true contact area on the pressure P on that element, which is of the form $S_T = S_0 p^h$ where S_0 and h are constant, and a general value of m, in the special model discussed above, are as follows:

When W is less than W_0, so that no asperities are completely flattened, the theoretical value of n will be that given by the theory of Lodge and Howell,[8] i.e.:

$$n = m + (1 - m)[m' + h(1 - m')] \tag{7}$$

where m and m' refer to the specimen and asperity deformation, respectively.

When W exceeds W_0 the equations, analogous to eqs. (3) and (4), that we obtain are (see Appendix II)

$$F = F_0 + (m/n)S_0[A(p_{\text{center.}})^h - A_0(p_0)^h] \tag{8}$$

and

$$F = F_0 + [(m\beta)^{(1-h)}S_0/\eta][W^n - W_0{}^n] \tag{9}$$

where

$$n = h(1 - m) + m \tag{10}$$

Writing as before in eq. (5),

$$F + \Delta = (m\beta)^{(1-h)}S_0W^n/n \tag{11}$$

where

$$\Delta = [(m\beta)^{(1-h)}/n]S_0W_0{}^n - F_0 \tag{12}$$

we see that, when F is much greater than Δ, F and W are related by a power law with index n which exceeds m if h is positive.

4. COMPARISON WITH EXPERIMENT

The experimentally observed variation of F with W (see Fig. 9 of the preceding paper) can be very closely represented by eq. (11) if we assume that the critical load W_0 was near the lowest load used and that h had the value 0.25 which is obtained by substituting the experimental values of n and m (i.e., 0.781 and 0.708) in eq. (10). The critical load W_0 can be crudely estimated from the departures from the observed friction which the power law shows when extrapolated to low loads. The estimates of W_0 for the fifteen specimens used in the measurements described in S4 of the preceding paper ranged from $\ll 0.7$ to about 1 g., tending to be greater for specimens of greater radius. From W_0, using eq. (A1)(Appendix I) and eq. (2) and the observed contact area, we can further deduce the critical pressure p_0 and the ratio C/R', i.e., the base radius to radius of curvature ratio for an asperity on the special model discussed in §3 and obtain values of about 0.7 kg./mm.^{2} for p_0 and 0.015 for C/R'.

Examination of the interference fringes between a specimen and the glass plate, especially the boundary of the zero-order fringe at low loads where the mutual inclination of the surfaces is least and the phase dispersion is greatest, showed that the height variations of the specimens were $\lesssim \lambda/20$, i.e., $\lesssim 0.027\ \mu$, but gave no clear indication of the lateral dimension of the asperities. If the asperity heights were as great as 0.027 μ and C/R_A were 0.015, the asperity diameter would be 7 μ.

The asperities on polished steel observed by Haine and Hirst,[14] using reflection electron microscopy, had maximum slopes and diameters similar to this, and one speculates whether the asperities on the polished steel balls are reproduced on the nylon specimens via the intermediate molds indented by the balls. Fidelity of reproduction of this sort can be shown by a single-stage replication with the use of polymethyl methacrylate and polystyrene, as Tolansky[15] has demonstrated.

In some measurements, preliminary to those described in the preceding paper, values of n of about unity were obtained for loads up to 20 g., the greatest that could then be used. The specimens used in these experiments were also of nylon 610, but the silver molds in which they were prepared were not gold-plated. The tangential tractions that were observed varied considerably up to a maximum of 0.8 kg./mm.^{2} It was in an attempt to reduce this scatter that the gold plating of the molds was introduced, in the belief that the more thorough cleaning processes that could then be used would result in greater reproducibility of the specimens. It now seems likely that tarnishing of the silver molds and, possibly, attack by the aqueous phenol used in cleaning them caused the specimens produced in the silver molds to have asperities which were not completely flattened under the greatest pressures employed. This would be consistent with the low tangential tractions that were observed. The observed value of unity for n in these experiments is also similar to the theoretical value based on the special model which, when W is less than W_0, gives $n = 0.94$ on substituting $m' = m = 0.708$ and $h = 0.25$ in eq. (7).

Confirmation of the significance of the asperity flattening pressure in determining the load at which the friction index n changes from that for a rough surface to that for a smooth one may be inferred from some published measurements by Archard.[6] Archard measured the friction F as a function of load W of crossed cylinders of Perspex (diameter 6.35 mm.) whose surfaces had been finished by turning. The pitch of the turning marks was 77 μ and their peak-to-peak height was 5–10 μ. At loads up to about 10 kg., F was proportional to $W^{1.0}$, but at greater loads F was proportional to $W^{0.71}$. For highly polished Perspex cylinders of the same diameter, F was proportional to $W^{0.71}$ for loads of 1–100 kg., and throughout this load range A varied as $W^{0.71}$. At a load of 10 kg. the apparent contact area was about 0.8 mm.^{2}.

Now, if we assume that the turning marks are parts of cylinders, their contacts, when the axes of the cylinders are perpendicular, will be equivalent to the contacts of spherical caps with C/R between 0.26 and 0.52 and the plane surface of a material of similar modulus. It can be shown that complete contact between this pair of surfaces will occur when the pressure reaches a value between 8 and 16 kg./mm.^{2}. These pressures are half those required to flatten similar spherical caps against a rigid plane, which is the circumstance for which eq. (8) was derived. (A value of 180 kg./mm.^{2}, which was obtained from the quoted deformation of the Perspex cylinders, was taken for $E/(1 - \sigma^2)$.)

In Archard's experiments the transition in the index n occurred when $W = 10$ kg. and $A = 0.8$ mm.^{2}. The central pressure was, therefore,

$$P_{\text{center}} = (1/m)(W/A), \qquad \text{i.e., } (1/0.71)(10/0.8), \qquad \text{i.e., } 17 \text{ kg./mm.}^2$$

The transition therefore occurred when the central pressure was sufficient to ensure complete contact between the two cylinders.

5. GENERAL DISCUSSION

We have seen that the observed variation of friction with load for the nylon specimens can be satisfactorily represented by the assumption that the asperities on the specimens are completely flattened at pressures of about 0.7 kg./mm.^{2}, if we additionally assume for elements of true contact area a dependence of specific friction S_T on pressure p of the form $S_T = S_0p^h$ where $h = 0.25$. The large range of loads, over which the relation $F = \alpha W^n$ holds with constant value of n, implies that no appreciable number of asperities exist which have critical pressures for complete flattening in the range that the central pressure takes for these loads, i.e., 0.6–6.5 kg./mm.^{2}. If there were lesser asperities that were never completely flattened and they deformed in a similar way to the specimens, then the arguments of §2.1 show that they could not lead to values of h less than m (i.e., 0.708). Therefore, we must assume that such asperities do not exist.

It seems likely, therefore, that the observed increase in specific friction with pressure requires, not a geometrical, but a molecular explanation, such as is required to explain the increase in bulk shear strength and in viscosity with pressure which is observed by Bridgman.[16] (The power-law relationship by which the specific friction dependence on pressure has been represented is not necessarily the physically correct one, for other relationships would probably be indistinguishable over the pressure range employed.) On a molecular model, such as that assumed by Bartenev[17] to interpret the velocity dependence of friction, the effect of a pressure increase could be to increase the potential barrier that must be surmounted by the molecular jumps.

The difference $n - m$ observed by Archard[6] at loads greater than W_0 was 0.05 ± 0.022 and is of low statistical significance, so that it does not provide reliable evidence of the dependence of specific friction on pressure. It is, however, nearer to the value observed here for nylon 610 (viz., 0.073 $\pm$ 0.0125) than it is to zero, and it would be consistent with the value of the index h, 0.25, found for nylon.

Some measurements made by Kraghelsky and Sabelnikov,[18] also on polymethyl methacrylate, show a strong dependence of specific friction on pressure. These authors used an optical method to measure the contact area between two roughened flat polymer surfaces (asperity heights up to 60 μ) for a 20–1 range of loads, and simultaneously measured the friction. They observed the mean specific friction to increase from 1.76 kg./mm.^{2}

at a mean pressure of 2.2 kg./mm.^{2}, to 4.86 kg./mm.^{2} at a mean pressure of 8.7 kg./mm.^{2}. Expressed as a power-law relation $\bar{S} = S(\bar{P})^h$, this gives $h \sim 0.7$, which is much greater than Archard's measurements would suggest. It is not clear, however, whether the contact area measured was the true or the apparent contact area. If it were the second and the gross asperities on the surfaces were themselves rough, i.e., covered by lesser asperities of, say, hemispherical shape, the observed value of h could be obtained even though the shear strength of the true contact area were independent of pressure. Interpretation is further complicated by the likelihood of interlocking of asperities on the two rubbing surfaces, since both were rough.

6. CONCLUSIONS

The load dependence of the friction of nylon 610 on glass, reported in the preceding paper, has been interpreted in terms of the adhesion theory of friction with the addition assumption that the shear strength S_T of the true contact area between nylon and glass increases slowly with pressure. In view of the interspecimen variability of friction, which probably indicates differences between the contaminating films on the rubbing surfaces, it is astonishing that such a simple interpretation should be possible. One is driven to conclude that the surface contamination was, at least to a first approximation, an independent variable just as speed and load were found to be.

In the light of the discussion of, and the extension to, the asperity theories of friction which have been given, it is possible to give the following description of the load dependence of friction, for elastic bodies, over a very wide range of loads.

At loads smaller than a critical load W_0, which causes complete flattening of the central asperities, the theories of Lodge and Howell and of Archard are appropriate. Under the lowest loads, where the approach of the rubbing bodies is small compared with the variation of asperity heights, Archard's treatment is applicable. When the approach becomes greater than the asperity height variation and the deformation of the substrate must be considered, we enter the range of applicability of Lodge and Howell's treatment. Both these theories predict values of n approaching unity and the effect of a pressure dependence of S_T makes the difference of n from unity even smaller.

At loads considerably greater than W_0 the rubbing surface is effectively smooth and the value of n is similar to that of the deformation index m, but exceeds it by an amount depending on the pressure dependence of S_T.

Estimates of W_0 can be made from feasible roughness measurements—for example, of the maximum inclination of the surface to its envelope—and can enable the effectively rough or smooth behavior of a surface under given conditions to be predicted.

A pressure dependence of the shear strength of the true contact area would be expected to influence the occurrence of abrasion when a polymer specimen slides on a smooth surface, for the following reasons. If the shear strength of the polymer is constant (or increases less rapidly with pressure than does the shear strength of the true contact area), it is conceivable that, under the high pressures which can exist at an asperity contact, the friction could be sufficiently great to detach fragments from the asperity without any permanent junction between the fragments and the mating surface. The presence of the fragments between the bodies would make the pressure distribution more irregular and could lead to the formation of more fragments.

Now, the pressures at the asperity contacts, even when the asperities are completely flattened, may be considerably greater than the average pressure on the apparent contact area if the asperities initially had large slopes. It would therefore seem valuable to correlate the average pressures at which abrasion commences for various surface textures of a polymer with, say, the maximum slopes of their asperities.

This work forms part of a program of fundamental research undertaken by the British Rayon Research Association.

Appendix I

The pressure $p_{\max}$ at the center of the contact of radius a between a Hookean sphere of radius R and a rigid plane produced by the application of a load W is (from Hertz[12]) given by:

$$p_{\max} = {}^3/_2 W/\pi a^2 \tag{A1}$$

and the pressure at radius r ($r < a$) from the center of the contact is given by:

$$p_r = p_{\max}(1 - r^2/a^2)^{1/2} \tag{A2}$$

so

$$p_r = {}^3/_2(W/\pi a^2)(1 - r^2/a^2)^{1/2} \tag{A3}$$

also[12]

$$a = [{}^3/_4 RW(1 - \sigma^2)/E]^{1/3} \tag{A4}$$

Eliminating W between equations (A3) and (A4) gives:

$$p_r = (2/\pi R)[E/(1 - \sigma^2)](a^2 - r^2)^{1/2} \tag{A5}$$

Now, the part of the contact subject to a pressure less than p_r is the annulus of area $\pi(a^2 - r^2)$ since the pressure decreases continuously from the center of the contact to the edge.

If we now consider the effect of increasing W whilst keeping p_r constant by suitably changing r we see from eq. (A5) that the area of this annulus will remain constant, for it depends only on the value of p_r and the radius and elastic constants of the sphere. Because this result is true for any value of p_r less than $p_{\max}$ it will be true when p_r equals the critical pressure p_0 (see §6) if pressures greater than this exist in the contact. Also, because this result is true for each of any pair of values of p_r (say, p and $p + \delta p$), the area of the annulus subject to pressures between p and $p + \delta p$ will be independent of W.

These conclusions will also be valid when the sphere is non-Hookean, i.e., when m in the equation $A = \beta W^m$ is not equal to $^2/_3$, if we write, in place of eq. (A2) above,

$$p_r = p_{\max}(1 - r^2/a^2)^{(1-m)/m} \qquad \text{(A6)}$$

whence

$$p_{\max} = W/m\pi a^2 \qquad \text{(A7)}$$

(Compare, for example, Equation 7 in Lodge and Howell.[3]) If we also replace eq. (A4) by:

$$a = [(K/\pi)^m W R^{2(1-m)}]^{1/2} \qquad \text{(A8)}$$

which follows from eq. (7) of the previous paper, the index of W in that equation, $2/(2 + x)$, being equal to m. Hence we obtain:

$$p_r = (1/m)\pi^{(1-m)/m} R^{-2(1-m)/m} K^{-1/m}(a^2 - r^2)^{(1-m)/m} \qquad \text{(A9)}$$

The deductions from eq. (A5) may similarly be made from eq. (A9).

Appendix II

We have shown in §3 and Appendix I that the contribution to the total friction made by the annulus of incomplete contact which borders the central area of complete contact is independent of the load W if $W \geqslant W_0$. This remains true even though the specific friction is pressure-dependent. We may, therefore, write for the total friction in this case:

$$F = F_0 + \int_0^{r_0} S(p) 2\pi r \, dr \qquad (W > W_0)$$

where $S(p)$ is the specific friction under the pressure p existing at radius r from the contact center, r_0 is the radius at which the pressure equals p_0 (the pressure at which asperities are just completely flattened), and F_0 is the total friction when r_0 is zero (which is when $W = W_0$) and is the contribution by the annulus of incomplete contact at higher loads.

Now, from eqs. (A6) and (A7), remembering that $\pi a^2 = A = \beta W^m$ we have:

$$p = (W^{1-m}/m\beta)(1 - r^2/a^2)^{1-m/m}$$

(No appreciable error is introduced by neglecting, as we do here, the small-scale variations of pressure which occur in a region where asperities have been completely flattened, provided that the central region of complete contact is large compared with the area of an asperity.)

Then, if we assume that $S(p) = S_0 p^h$ we have:

$$\begin{aligned} F &= F_0 + S_0 \int_0^{r_0} [(W^{1-m}/m\beta)(1 - r^2/a^2)^{1-m/m}]^h \, 2\pi r \, dr \\ &= F_0 + (W^{1-m}/m\beta)^h S_0 [m\pi a^2/(h(1 - m) + m)] \\ &\qquad \{1 - (1 - r_0^2/a^2)[h(1 - m) + m]/m\} \end{aligned} \qquad \text{(A10)}$$

Now, by definition, $p(r_0) = p_0$ and $(p_{\max})_{W=W_0} = p_0$, therefore:

$$(W^{1-m}/m\beta)(1 - r_0^2/a^2)^{1-m/m} = (W_0^{1-m}/m\beta)$$

$$\text{i.e.,} \qquad 1 - r_0^2/a^2 = (W_0/W)^m$$

Remembering again that $\pi a^2 = \beta W^m$ and introducing the parameter $n \equiv h(1 - m) + m$, eq. (A10) becomes:

$$F = F_0 + (m\beta)^{1-h} S_0 (W^n - W_0^n) \quad \text{(A11)}$$

Otherwise, since $p_{\max} = W^{1-m}/m\beta$ and $p_0 = W_0^{1-m}/m\beta$,

$$F = F_0 + m/n S_0 [A(p_{\max})^h - A_0 p_0^h] \quad \text{(A12)}$$

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Synopsis

It was concluded in the preceding paper that there was a significant difference between the indices m and n in the expression $A = \beta W^m$ and $F = \alpha W^n$ by which the contact area A and the friction F were found to be related to the load W when nylon 610 slid on glass. A discussion of published adhesion theories of the friction of elastic bodies shows that the difference $n - m$ is too small to be attributed to roughness of the nylon surface. It is concluded that under the contact pressures used the nylon surface was effectively smooth and that the friction of a unit area of "true" (i.e., molecular) contact between nylon and glass increased with pressure P as about $P^{1/4}$. A theoretical treatment is given of the dependence of friction on load for a rough-surfaced elastic body which covers the load range in which the surface asperities become completely flattened. It is shown that from measurements of the elastic properties of the body and feasible measurements of its surface texture one can estimate the load at which the dependence of friction on load changes from that for a rough to that for a smooth surface. Comparison with a published experiment is satisfactory.

Résumé

Dans l'article précédent, on a conclu qu'il y avait une différence considérable entre les indices m et n dans l'expression $A = \beta W^m$ et $F = \alpha W^n$ où la région de contact A et la friction F semblent être reliées à la charge W quand le nylon 6.10 glisse sur du verre. Une discussion des théories d'adhésion publiées sur la friction de corps élastiques montre que la différence $(n - m)$ est trop petite pour être attribuée à la rugosité de la surface du nylon. On en conclut dès lors que la surface du nylon était effectivement peu rugeuse sous les pressions de contact employées et que la friction d'une région d'unité de contact réel (c.à.d., moléculaire) entre le nylon et le verre augmentait en fonction de la pression P à la puissance $^1/_4$ environ. On donne un traitement théorique de la dépendance de la friction de la charge pour une substance élastique avec une surface brute qui couvre le domaine de charge dans lequel les aspérités de la surface s'applattissent complètement. On montre qu'à partir de mesures des propriétés d'élasticité du corps et de mesures possibles de sa texture de surface, on peut évaluer la charge pour laquelle la dépendance de la fraction aux changements de charge varie de la valeur pour une surface brute à la valeur pour une surface lisse. La comparaison avec des expériences publiées est satisfaisante.

Zusammenfassung

In der vorhergehenden Arbeit wurde ein charakteristischer Unterschied zwischen den Exponenten m und n im Ausdruck $A = \beta W^m$ und $F = \alpha W^n$ für die Beziehung der Kontaktfläche A und der Reibung F zur Belastung W beim Gleiten von Nylon-6-10 auf Glas gefunden. Eine Diskussion der bekannten Adhäsionstheorien der Reibung elastischer Körper zeigt, dass die Differenz $(n - m)$ zu klein ist, um der Rauhigkeit der Nylonoberfläche zugeschrieben zu werden. Es wird daher angenommen, dass bei dem angewendeten Kontaktdruck die Nylonoberfläche effektiv glatt war und die Reibung der Flächeneinheit "wahren" (d.h., molekularen) Kontaktes zwischen Nylon und Glas mit dem Druck P etwa nach $P^{1/4}$ anstieg. Eine theoretische Behandlung der Abhängigkeit der Reibung von der Belastung für einen elastischen Körper mit rauher Oberfläche wird für den Belastungsbereich, in welchem die Oberflächenrauhigkeiten völlig geglättet werden, gegeben. Aus Messungen der elastischen Eigenschaften des Körpers und durchführbaren Messungen seiner Oberflächentextur kann man die Belastung bestimmen, bei welcher die Abhängigkeit der Reibung von der Belastung von einer solchen für eine rauhe zu der für eine glatte Oberfläche gültigen übergeht. Der Vergleich mit einem veröffentlichten Versuch ist befriedigend.

Received September 10, 1962